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NUMBER THEORY.

218. Proposed by ELIJAH SWIFT, University of Vermont.

If p is a prime >3, show that $\sum_{a=1}^{p-1}\frac{1}{a^2}\equiv 0 \pmod{p}$.

II. Solution by the Proposer.

It may be proved (Bachmann, *Niedere Zahlentheorie,* page 155) that $A_{p-3} = 1\cdot 2\cdot 3\cdots p-3+1\cdot 3\cdot 4\cdots p-2+\cdots$, taking all possible combinations of the numbers $1, 2, \cdots p-1$, $p-3$ at a time, is divisible by p.

Then

$$A_{p-3} = (p-1)!\sum_{\substack{a=1\\b=1}}^{\substack{a=p-1\\b=p-1}}\frac{1}{a\cdot b} \qquad a \neq b.$$

Then

$$\sum\frac{1}{a\cdot b}\equiv 0 \pmod{p}.$$

This sum may be written

$$\frac{1}{2}\left\{\sum_{b=1}^{p-1}\frac{1}{1\cdot b}-\frac{1}{1^2}+\sum_{b=1}^{p-1}\frac{1}{2\cdot b}-\frac{1}{2^2}+\cdots+\sum_{b=1}^{p-1}\frac{1}{(p-1)b}-\frac{1}{(p-1)^2}\right\}.$$

But

$$\sum_{b=1}^{p-1}\frac{1}{a\cdot b}\equiv\sum_{b=1}^{p-1}\frac{b}{a}\equiv\sum_{b=1}^{p-1}b\equiv 0 \pmod{p}.$$

Hence,

$$\sum\frac{1}{ab}\equiv-\sum_{a=1}^{p-1}\frac{1}{a^2}\equiv 0 \pmod{p}.$$

QUESTIONS AND DISCUSSIONS.

Edited by U. G. Mitchell, University of Kansas.

NEW QUESTIONS.

26. Why should not the nomenclature of mathematics be made uniform? For example, why call a circle a *portion of a plane* in elementary geometry and a *curve* in analytic geometry? Why call a sphere a *ball* at one time and a *surface* at another time? And so on through all the configurations of two- and three-dimensional geometry.

REPLIES.

18. In view of the present pressure for saving time and gaining efficiency, what are the most important sources of economy in the mathematical courses of the high school and the first two years in college?

Reply by Ernest W. Ponzer, Leland Stanford Junior University.

This question assumes the existence and application of the fundamental principles of efficiency[1] in connection with our college instruction in mathematics. No doubt the editors are aware of the fact that many college professors would hardly make this assumption and that many others would regard the problem involved as quite trivial. If such is not the case, why, for instance, in many large universities is instruction in the mathematics of the freshman year given to a majority of the students enrolled in these courses by assistants in the department? And why is the ferment for a square deal to those courses now working so strongly among those vitally interested, the students themselves and the people who support the institutions? We well know that any assumption of high efficiency in this work is an assumption contrary to fact and that we shall be held responsible for our methods and our results.

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[1] Cf. article "A Study in Efficiency," *School Science and Mathematics,* October, 1910, pp. 579–81.

Shall we assume the problem to exist, to be worth while, and not to be regarded as a chore in an otherwise interesting round of labors? We must and do; in fact, we believe it to be a fundamental problem in the building of a solid foundation for the mathematical education of the student.

To obtain efficiency we suggest starting with the instructor. We do not believe that the instruction of the first two years should be given by assistants with little or no experience in teaching the subjects. It is worthy of the best efforts of the ablest and most mature instructors in the department. We suggest a recognition of the value of this work and we would accord credit (and salary) to show our appreciation when it is efficiently done. The instructor's attitude toward his work should first of all be sympathetic and he should have the courage of his convictions in carrying it out.

The instructor should have good raw material on which to work. While he must take all who come with the proper credentials, he can eliminate the weaker students. At Stanford for a number of years we have eliminated each fall from our first year sections in applied mathematics on an average about ten per cent. of the students registered. This is done after a review of secondary algebra extending over about three weeks concluded by an examination. The ten per cent. were certainly not at that time up to the standard required for efficient work. Those eliminated had to get a better preparation. Their tales of why they could not at that time hope to handle their mathematics as they should are interesting tales of inefficient work done in high schools. You have all heard these. The stimulus of the sifting process on the ninety per cent. remaining is great. They have drawn a bonus and are alert for more. Their attitude toward their work is excellent. The instructor can begin to see the gradual working of the refining process which is to extend for a year through algebra, review of trigonometry, and analytic geometry, with perhaps the elements of the calculus. Sections now average about 30—small enough for individual and laboratory work and not large enough to require a lecture treatment. Better have fewer good instructors and more good student assistants than many teaching assistants.

We would have student assistants to read papers and exercises handed in. Necessarily these are handed in on time—likewise returned on time. A schedule of assignments based on the text in use is early placed in the hands of students. And they are held responsible for their share in an intelligent discussion of the assignments from day to day.

Perhaps the methods of the instructor in conducting the courses should vary with the class of students enrolled. My experience has been almost exclusively with classes whose major interest was in applied science, especially those electing engineering as their major. An instructor should acquaint himself with the aims of his particular classes of students and then adapt his methods to suit their needs. For instance, and for applied mathematics: "The problems of applied science call for quantitative results. Abstract and formal results are not sufficient. They must be correct to the proper decimal point. They should check. Accuracy and efficiency must obtain. Form and logical sequence must be seriously considered. Suitable methods whether graphical, numerical, or mechanical are necessary. The province of applied mathematics is to furnish such suitable methods. And the field is not at all to be considered as apart from pure mathematics but a part of it. It is simply a question of a different state of mind handling its problems efficiently"—I quote (translated) from Runge, of Göttingen, Germany.

A teaching assistant whose main interest is centered on the courses in advanced work in mathematics which he himself is taking will too often fail to hit the nail on the head. Efficiency in instruction will too often be low. And no one is deceived by the procedure—least of all the freshman student who is laying a foundation for his future work.

Early in the work of the first year; in fact during that of the first semester, I have made it a practice to insist on the checking of results by the aid of the slide rule. At the end of the year all handle this phase of the work efficiently. This checking is fundamental, that is the way the engineer works; why not use this tool early? Elaborate checks, such as are frequently given in the solution of triangles in trigonometry, are quite unnecessary to the efficient student who applies his common sense and his slide rule as he goes. A similar word might be said for the planimeter which naturally is used more in connection with the calculus, not to mention many other mechanical aids and tables. These are not to be used for first solutions but as efficient aids in checking. Since the work is thus done where mathematics is applied, why should we fall so far short in our instruction? A fair trial will convince the most skeptical or indifferent instructor of the value of all such aids.

I have confined my remarks mainly to the work of a first year course, for I believe that the instructor can there either make or mar, to a large extent, the work of the student in the later years of his course. Certainly he can greatly influence its efficiency. And the student certainly has a right to expect that he will get a square deal from the beginning instead of having the courses he takes treated with indifference by those in charge of them.

I would call for high grade and efficient instruction all along the line. And the best feature of it all is that the students will appreciate the standards set and the efforts of all of us toward securing this higher efficiency.

3. In connection with the theory of the conduction of electricity through gases, one is led to the differential equation

$$y\frac{d^2y}{dx^2} + a\left(\frac{dy}{dx}\right)^2 + b\frac{dy}{dx} + cy + d = 0, \tag{1}$$

where a, b, c, d are constants. For unrestricted values of a, b, c, d the solution of this differential equation presents peculiar difficulties, the series solutions obtained by the customary methods having (apparently) too small a range of convergence to be satisfactory from the point of view of electrical theory. The general solution of this equation is wanted in case it can be found. If no general solution is obtained for unrestricted a, b, c, d, it is desirable to know special values of a, b, c, d or special relations among a, b, c, d which make it possible to find the general solution; and this solution is desired in each case.

REPLY BY W. W. BEMAN, University of Michigan.

The given equation may be written

$$yy'' + ay'^2 + by' + cy + d = 0.$$

Put $y' = p$. Whence $y'' = p\dfrac{dp}{dy}$.

Then

$$yp\frac{dp}{dy} + ap^2 + bp + cy + d = 0$$

and

$$\frac{pdp}{cp^2 + bp + cy + d} + \frac{dy}{y} = 0.$$

If c be 0, the variables are separated. If now $p = f(y)$ or $y = \phi(p)$, we may be able to integrate again, expressing y as a function of x, or x and y as functions of p.

It is interesting to note that the integration called for here in terms of y and p is the same as the one involved in the solution given in the February number of the MONTHLY (page 72) when $a = 1$.

Also, PROFESSOR C. E. LOVE of the University of Michigan makes the following contribution toward the solution of this equation.

Writing the equation, for greater distinctness,

$$yy'' + \alpha y'^2 + \beta y' + \gamma y + \delta = 0$$

and taking y as the independent variable,

$$yx'' - \alpha x' - \beta x'^2 - (\gamma y + \delta)x'^3 = 0.$$

If $\beta = 0$, we have the linear extended form. Whence

$$\frac{1}{x'^2} = \frac{-2\gamma y}{2\alpha + 1} - \frac{\delta}{\alpha} + Cy^{-2\alpha},$$

$$x = \int \frac{dy}{\sqrt{-\dfrac{2\gamma y}{2\alpha + 1} - \dfrac{\delta}{\alpha} + Cy^{-2\alpha}}}$$

which can be evaluated if $\alpha = -1$ or $\alpha = -\frac{1}{4}$.